A Better Match Of The EGM96 Harmonic Model For The Egyptian Territory Using Collocation

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Abstract:

It is well known that using a global geopotential model, as a reference field, is crucial for a high quality local (or regional) gravimetric geoid determination. However, the application of this technique would only be reliable, if the region under investigation has gravity data contribution to the used harmonic model. EGM96 represents the most recent high degree global geopotential model that is completely released to the geodetic community. However, during the solution for that model, no Egyptian terrestrial gravity data were taken into account. This paper presents an attempt to locally refine the EGM96 harmonic model to fit Egypt better. Using least-squares collocation technique, corrections for the EGM96 spherical harmonic coefficients along with their error estimates are predicted based on the recent available gravity data in Egypt. Using the refined EGM96 model, which will be denoted as EGM96EGR, as a reference field showed an improvement in gravity anomaly smoothing by about 40%, which represents a great efficiency in low frequency gravity field modeling.

1 Introduction

Recently, the remove and restore technique of a global high frequency geopotential model is inevitable in most of the new approaches of determining the geoid locally. Beside providing long wavelength information and reducing the truncation error (Amin, 1983), the subtraction of a global field from the local data results in a residual smooth field that is essential for an accurate and precise gravity field signals prediction by collocation. However, the low frequency part removal and addition would only be realistic and meaningful, if the model contains local gravity information from the region under study (Hanafy, 1993; El-Tokhey, 1995; El-Sagheer, 1995 and Nassar et al., 2000). EGM96 is the most recent high degree global geopotential model that is completely available for the geodetic community. Containing new and increased satellite data only, high-resolution terrestrial gravity data and altimetry data, EGM96 is superior over the other preceding models (Lemoine et al., 1996). The accuracy of geoidal heights computed from EGM96 could be as well as a few decimeters in areas having data contribution to it (Smith and Milbert, 1997). On the other hand, the accuracy in regions with no data contribution to the model could go down to 2-3 meters (Smith, 1998).

In fact, regarding Egypt, the EGM96 model suffers, as all as the other geopotential models, from the absence of the terrestrial gravity data (Amin, 2002). Consequently, a

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considerable long wavelength error will certainly accompany the process of the remove and restore of that model. Thus, the long wavelength information for the Egyptian territory cannot be optimally recovered from such global model.

The aim of this study is to refine the EGM96 model using the available Egyptian data. Utilizing the least-squares collocation (LSC) technique, the functional relationship between the anomalous potential and the spherical harmonic coefficients was exploited to predict corrections for these coefficients based on the local Egyptian gravity data. The resulting refined model, denoted by EGM96EGR, was evaluated and showed that a strong improvement has been achieved over the original model with respect to the Egyptian territory.

2 Covariance between the harmonic coefficients and the disturbing potential

The LSC technique as a general mathematical method is considered one of the best techniques for determining any parameter of the Earth's outer gravity field, on the basis of the statistical relationships that exist between these actual parameters. This method may also be used to predict what is called signals, which may exist at stations other than the data points, e.g. at grid points. These statistical relationships are manifested in the so-called covariance function. However, if the spherical harmonic coefficients of the anomalous potential are to be predicted (as signals), we will need the explicit covariances between the harmonic coefficients and the anomalous potential, T, or more practically any of its observable functions, such as gravity anomalies, geoid undulations, deflections of the vertical, etc. (Tscherning, 2001). In what follows, a compact overview of these relations will be outlined.

Let P and Q be two points with coordinates (φ,λ,r) and (φ',λ',r'), respectively, and having spherical distance ψ. If R is the mean radius of the Earth, P_i the Legendre polynomials and σ_i^2 the potential degree variance. Then, the covariance between the values of the anomalous potential T in P and Q is (Heiskanen and Moritz, 1967)

$$\mathrm{cov}(P,Q) = \sum_{i=2}^{\infty} \sigma_i^2 \,.\, (R^2/r.r')^{i+1} \,.\, P_i(\cos\psi)$$

$$= \sum_{i=2}^{\infty} (\sigma_i^2/(2i+1)) \,.\, R^2 \sum_{j=-i}^{i} (R^i/r^{i+1})\overline{Y}_{ij}(\varphi,\lambda) \,.\, (R^i/r'^{i+1})\overline{Y}_{ij}(\varphi',\lambda'), \qquad (1)$$

where $\overline{Y}_{ij}(\varphi,\lambda)$ are the fully normalized surface harmonics, which are given by

$$\overline{Y}_{ij}(\varphi,\lambda) = \overline{P}_{i|j|}(\sin\varphi)\ \sin|j|\lambda, \qquad j < 0$$

$\bar{Y}_{ij}(\varphi,\lambda) = \bar{P}_{ij}(\sin\varphi) \cos j\lambda, \quad j \geq 0.$

Analogous to the above convention for the surface spherical harmonics, let $\bar{K}_{ij}$ be a general symbol for a fully normalized (unitless) spherical harmonic coefficient of degree i and order j, so that (Tscherning, 1974)

$$\bar{K}_{ij} = \bar{S}_{ij} \qquad j < 0,$$

$$\bar{K}_{ij} = \bar{C}_{ij} - \bar{J}_{ij} \qquad j = 0 \text{ and } i \text{ even},$$

$$\bar{K}_{ij} = \bar{C}_{ij}, \qquad j > 0, \text{ or } j = 0 \text{ and } i \text{ odd}.$$

where $\bar{J}_{ij}$ are the relevant normal zonal harmonic coefficients induced by the reference mean Earth ellipsoid.

The covariance between the harmonic potential coefficient $GM.\bar{K}_{ij}/R$ and the anomalous potential is obtained by applying its functional operator, L_{ij}, on the covariance function, where

$$L_{ij}(T) = (1/4\pi R^2) \iint_e \left((GM/R). \sum_{n=2}^{\infty} (R/r)^{n+1} \sum_{m=-n}^{n} \bar{K}_{nm}\, \bar{Y}_{nm}(\varphi,\lambda).(R/r)^{i+1}.\bar{Y}_{ij}(\varphi,\lambda)\right).$$
$$R^2. \cos\varphi \, d\varphi \, d\lambda$$
$$= (GM/R).\bar{K}_{ij} \qquad (m^2/s^2), \tag{2}$$

where the integration is carried out over the whole globe. Thus, applying this function on the anomalous potential covariance function, cov(P,Q), it can be proved that the target covariance function is given as

$$\mathrm{cov}((GM/R).\bar{K}_{ij}, T_Q) = L_{ij}(\mathrm{cov}(P,Q))$$
$$= (\sigma_i^2/(2i+1)).(R/r')^{i+1} . \bar{Y}_{ij}(\varphi',\lambda'). \tag{3a}$$

This expression states that the covariance between the anomalous potential at the point Q and the scaled (non unitless) harmonic coefficient is simply a function of the relevant potential degree variance and the related solid spherical harmonic function evaluated at Q. Applying the functional operator once again, one obtains the function's variance (Tscherning, 2001), namely

$$\mathrm{cov}((GM/R).\bar{K}_{ij}, (GM/R).\bar{K}_{ij}) = L_{ij}(L_{ij}(\mathrm{cov}(P,Q)))$$
$$= \sigma_i^2/(2i+1). \tag{3b}$$

Eq.(3b) is simply the auto-covariance of the coefficient $(GM/R).\bar{K}_{ij}$ with itself. It is well known that its cross-covariance with any different coefficient is by definition equal to zero, due to the orthogonality relationships among the spherical harmonic coefficients (Heiskanen and Moritz, 1967).

Using the law of covariance propagation, the covariance with an arbitrary (observed) gravity field function, L_g, is given by

$$\mathrm{cov}((GM/R).\bar{K}_{ij}, g_Q) = L_g\,(L_{ij}(\mathrm{cov}(P,Q)))$$

$$= (\sigma_i^2/(2i+1)).\ L_g\,((R/r')^{i+1}\ .\bar{Y}_{ij}(\varphi',\lambda')), \qquad (4)$$

or simply, the functional operator, L_g, is directly applied to the relevant solid spherical harmonic. This procedure can then be used for the evaluation of the cross-covariance functions between a specific (to be predicted) harmonic coefficient and any point data during the LSC prediction process. As can be easily shown above, these cross-covariance functions are functions only of the respective degree and order, the (potential) degree variance and the spatial location of the data point under consideration.

3 Viewing the missing local data long wavelength contribution as a "correction" to be applied to the original harmonic model

Let's first imagine that the local terrestrial gravity data has been incorporated into the global solution for the high degree and quality EGM96 global harmonic model. As a consequence, the EGM96, as a reference field, would recover the low frequency spectrum in a reliable manner during a local geoid solution for Egypt, which would be obvious from the great smoothness of the used data after the remove process of the EGM96 model. This smoothing effect represents a theoretical necessity for treating the anomalous potential and its functions as spatially random signals with possibly very small spatial mean signal and standard deviation. This would reflect itself on the statistics as well as on the empirical covariance function of the residual data, when using LSC technique. In this case, the residual data, after accounting for the topographic effect, could be safely assumed to be of a medium-to-high frequency nature (Shaker et al., 1997). Thus, the model behavior in the relevant region would be surely tuned to reflect the characteristics of the collected local data, which represents the best gravity field information for its region. Consequently, the resulting referenced local geoid solution would be of a better quality.

However, as it is the case in Egypt, if the region under study has no data contribution to a global geopotential model, one is certainly faced with an opposite situation. Namely, only the global satellite data inherent to the model would be turned on during the referencing of the local data and the solution to such a model. This very long wavelength effect, which for EGM96 is as high as degree and order 70, although realistic, it is not sufficient to achieve the required geoid quality for modern geodetic requirements. In such regions, however, low frequency residual data and residual solutions with unrealistically large power would result. In other words, one obtains nonzero "residual" low degree information. Such residual information may be miss-interpreted and handled as higher frequency one. Hence, it would be accompanied with a long wavelength error and would deteriorate the required quality, even if the local data has good coverage and resolution. The harmonic model would only succeed in suppressing a small amount of lower degrees that approximates the satellite-derived spectrum inherent into it, thus not realizing its (intended) nominal maximum resolution (or degree). So, until the local data could be internationally introduced to such a high quality model or newer released versions, a long wavelength correction for EGM96 is attempted in the current study based on the available local data set.

4 Using the remove-restore technique to account for the "residual" long wavelength spectrum

Based on the above discussion, one would agree that incorporating the local data into the raw global data model before its establishment is nearly equivalent to use the current model in a remove-restore technique and predict an equivalent set of harmonic coefficients corrections. Particularly, if the input data are geoidal heights, N, then the EGM96 long wavelength geoid component is removed to obtain the residual geoidal data,

$$N_{res} = N - N_{EGM96}, \tag{5a}$$

where

$$N_{EGM96} = (GM/r\gamma) \sum_{n=0}^{360} (a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\theta), \tag{5b}$$

with

θ	the geocentric latitude,
λ	the geodetic longitude,
r	the geocentric radius to the geoid,
$\gamma(\theta,r)$	the normal gravity induced by the WGS-84 reference ellipsoid,
GM	the Earth mass-Gravitational constant product consistent with the EGM96 coefficients,
a	the equatorial radius scale factor associated with the EGM96 model,
$\bar{C}^*_{nm}$	the EGM96 fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the WGS-84 reference ellipsoid,
$\bar{S}_{nm}$	the EGM96 fully normalized spherical harmonic S-coefficients of degree n and order m,

$\bar{P}_{nm}(\sin\theta)$ the fully normalized associated Legendre function of degree n and order m.

The obtained N_{res} values are then used as input for the LSC solution to receive the harmonic coefficients corrections, $(GM/R).\Delta K_{ij}$. The unitless coefficients' corrections, ΔK_{ij}, are then restored (added back) to the EGM96 relevant coefficients, in order to end up with the EGM96EGR coefficients, $K_{ij\ EGM96EGR}$,

$$\bar{K}_{ij\ EGM96EGR} = \bar{K}_{ij\ EGM96} + \Delta K_{ij}. \tag{5c}$$

Our intended procedure could be logic because in this manner, the residual local long wavelength spectrum not accounted for by the original model, should be treated in such a way so as to incorporate their possibly (local) significant terms into that model. These (low frequency) predicted corrections are based on the local residual data that contains combined residual low and high frequency information. This is evident as long as the used data are point values, which theoretically contain spectral information up to infinity, provided the region under study (and the whole globe around it) is continuously covered with gravity data. In this ideal case, the Earth's gravity field would be full determined and there would be no need for any statistical tools, and a global geopotential model of maximum degree infinity could be achieved theoretically (Tscherning, 1974). However, as this fictive data coverage continuity is impossible, the point data would produce a maximum spectral limit according to its nominal mean density and distribution.

One should not worry about the eventual forcing of the coefficients corrections to carry higher frequency information inherent to the local data, since there is a one to one correspondence between the harmonic coefficients-anomalous potential cross covariance functions and their respective degree & order and their modeled degree variances. It is easy to show that because the degree variances in that remove procedure is considered proportional to the original coefficients error degree variances (Eq.(6)), the coefficients that are assumed to be errorless would not receive a correction. This means that the resulting harmonic coefficients corrections and (their error estimates) would depend on the standard deviations (noise) of relevant original coefficients, reflected by the covariance function model, the data noise, the data coverage and resolution. This emphasizes the principal property of a LSC solution that produces the same predicted value for an errorless observation and an estimate within the "error-band" of a noisy observation.

On the other hand, the theoretical rule of thumb that e.g. 0.5ºx0.5º mean data would contain exactly 180º/0.5º maximum degree information, although instructive and used internationally, is based on the supposition that the data are errorless (Meissl, 1971). Even if this were true, such a grid of mean gravity anomalies would contain spectral information greater than 180º/0.5º, due to the deviation of the grid block mean from the mean based on an area-equivalent circular cap (Rapp, 1977).

5 Data organization

According to the above, and based on the flexibility of the available algorithm to deal with point and mean data, it was intended to have a compromise for the data organization prior to its use to predict local based corrections for EGM96. The original irregular point free air gravity anomaly data was initially used to solve for a 0.5ºx0.5º (point values) free air geoid grid covering the Egyptian territory, based on EGM96. The (LSC) free air geoid solution was computed relative to the WGS-84 reference ellipsoid.

This (residual) geoid grid was then used for the estimation of the harmonic coefficients' corrections as well as their error estimates. It also exploits the advantage of having an even distribution of the used data with a reasonable number of data points that represent, along with their LSC estimated noise, the original data effectively. It was essential to have so a limited number of (estimated) data, since the LSC estimation of the coefficients corrections, as well as their error estimates, is very time consuming. The geoidal height is referred to as the smoothest version of all anomalous potential functions (Meissl, 1971). This is evident, due to the fact that the geoid in general has most of its power in the low degree (slowly spatially varying) spectral band. Conversely, the (less smooth) gravity anomalies, vertical deflections, etc., are poor in the lower degree information and the higher frequency spectral bands constitute the majority of its power spectrum. As the aim was to estimate harmonic corrections of low degree nature, the geoid was suggested to represent the most appropriate start data type to recover this information. Table (1) shows the statistics of the input free air geoid grid along with its standard deviations.

Table (1) Statistics of the input free air geoid grid (483 points) (units: meters)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Geoidal height	13.85	2.90	14.15	6.82	20.94
Residual geoidal height	0.02	0.95	0.95	-2.66	5.26
Standard deviation	0.70	0.17	0.72	0.25	0.89

6 Modeling the local covariance function

It is well known that the LSC procedure requires the estimation of the isotropic empirical covariance function of the residual data. This isotropic covariance function, which is a function of the separation between the data points, describes the spatial variability of the local residual field under consideration. The main features of this function are the variance (covariance at zero distance), the radius of curvature of the covariance function curve at the same point and the correlation length, which corresponds to a positive covariance value that is equal to half the variance. Of great importance is the formulation of the model (analytical) covariance function that is best fitted to the empirical one in a least-squares sense. The model covariance is uniquely described through three parameters as well be clarified below. In what follows, the general considerations and possibilities for modeling the analytical covariance function are outlined.

6.1Generalconsiderations

Generally speaking, a modeled local covariance function that is consistent with the remove-restore of the original model is used to account for the removed spectrum. A degree variance that is equal to a scaled error degree variance for the used model is used up to the maximal degree of that model. The higher frequency degree variances are modeled with a well-established degree variance model (Tscherning and Rapp, 1974). The modeled (analytical) covariance function is fitted to the residual anomaly empirical covariance function via a nonlinear 3-parameter iterative least-squares adjustment. The local isotropic anomaly covariance function model can be given as (Tscherning, 1993)

$$C(P,Q)= C(r,r',\psi)$$

$$=\sum_{n=2}^{nmax} c.\sigma^2_{neEGM96}.(R_b^2/rr')^{n+2}P_n(\cos\psi)+\sum_{nmax+1}^{\infty} A.(n-1)/(n-2).(n+24).(R_b^2/rr')^{n+2}.P_n(\cos\psi), \quad (6)$$

where

ψ	the spherical distance between the two points P and Q,
r	the geocentric radial distance of point P $\approx R+H_P$,
r'	the geocentric radial distance of point Q $\approx R+H_Q$,
R	the mean radius of the Earth, taken $\approx$ 6371 km,
R_b	the radius of the Bjerhammar's sphere,
$\sigma^2_{neEGM96}$	the n^{th} anomaly error degree variance based on EGM96 coefficients' standard errors,
c	a positive unitless scale factor,
A	a positive constant ($mgal^2$),
N_{max}	360 (max degree of EGM96),
H	orthometric height of the respective point.

The three parameters c, A and (R_b-R) are given firstly approximate values. The adjustment is then performed in an iterative manner until the convergence is arrived, resulting in the final three parameters c, A in $mgal^2$, (R_b-R) in meters and the point gravity anomaly variance at MSL as a by-product. These parameters are determined, based on the local residual gravity anomaly data, via its empirical covariance function. The final values are then used as input for the collocation process. In the LSC solution stage, the law of covariance propagation is executed to account for all possible varieties of auto-and cross covariances related to the observed functions during the solution (Tscherning and Rapp, 1974). For the special case of harmonic coefficients prediction, Eq.(4) is used to account for cross-covariance functions between the coefficients (corrections) to be predicted and any possible observed anomalous field functions.

It is worth mentioning that the scale factor "c" is a measure of how well a specific harmonic model fits the local data low frequency information. In other words, if the model were absolutely consistent with the data low degree part (i.e. had an optimally realized local data contribution), one would have a zero scale factor. Intuitively, a value of unity for this scale factor could signal that the model achieves its globally estimated accuracy in the region under study. Hence, this value can be assumed to be the maximum value behind which the model could be judged not to fit the region low frequency information well. Thus, in general, the better the model recovers the local data long wavelength spectrum, the smaller is the scale factor, c, than unity and vise versa. Consequently, a (possible) miss-modeling of the low frequency part of the data by the global model is translated into a magnification of the model's error spectra in a LSC solution and, vise versa. Hence, the so modeled local covariance function will result in LSC predictions and error estimates that truly mirror the model accuracy in the region under study. This will be of course accompanied with the effect of the local residual field variation expressed by its variance, data coverage, resolution and noise.

6.2 Possible procedures for covariance function modeling

In our special case of harmonic coefficients corrections estimation, a great attention should be paid to covariance function modeling, since this item (among other factors) is crucial for the resulting predictions along with their error estimates. As will be mentioned, the modeling of that function should be consistent with the target task, in order to obtain a meaningful solution for the aimed harmonic model corrections.

If a harmonic model possesses perfect lower degree terms that are consistent with the local data, then these coefficients should be left intact. This can be accounted for by assigning zero error degree variances for these coefficients in the first term of Eq.(6) and in Eq.(3,4). Hence, these coefficients would have zero corrections with zero error estimates, thus realizing the goal of keeping these terms fixed. The corrections (and error estimates) are then only obtained for the rest of the harmonic model coefficients, based on the rest of the error degree variances and the input residual data. Particularly, if the first zero point of the residual data empirical covariance function is located at spherical distance ψ^o, then the original model has practically removed $180^o/\psi^o$ lower degree terms from the local data (Meissl, 1971). Again, the corresponding error degree variances could be assumed zero for the removed terms in the covariance function model, thus fixing the corresponding coefficients in the LSC procedure. As above, the remaining coefficients could now receive corrections and error estimates, based on the relevant error degree variances and the input data. Also, this trend would be elegant, if the aim is to locally extend the resolution of a new satellite only harmonic model, e.g. EGM96S (70,70), based only on the Egyptian local gravity data.

However, as it is the case with modern high degree global harmonic models, it is well known that EGM96 had firstly a combined low degree solution up to degree and order 70. Namely, the up to 70 degree and order satellite only coefficients were merged with global $1^o x 1^o$ mean gravity anomalies in a further enhanced combined low degree solution again up to degree and order 70. The latter solution was then merged with global $0.5^o x 0.5^o$ mean gravity anomalies in a high-resolution solution up to degree and order 359, and finally, it was solved for the terms of degree 360 (Lemoine et al., 1996). Concerning Egypt, in each of these combined solutions, the Egyptian data should have been incorporated in the global data base. Thus, as far as local corrections for a high degree global model are concerned, it would be meaningful to seek for local corrections for all the EGM96 coefficients (from degree 2 to 360) to compensate the absence of the local data during the global solution steps for the EGM96 model. This requires no special treatment for covariance function modeling. Thus, the general discussion in Section 6.1

will still be valid for this task. The EGM96 corrections will depend mainly on the error degree variances of the first summation in the right hand side of Eq.(6) and in Eq.(3,4) along with the input residual data. This strategy is used in our current investigation.

Moreover, if one is interested in the estimation of harmonic coefficients of degree higher than 360, then the second summation in Eq.(6), also accompanied with Eq.(3,4), would be the dominant one, since it already contains (non zero) modeled degree variances. However, the actual spectral content inherent into the data will judge the maximum degree and order of the significant coefficients that could be extracted.

7 Computations

In the current work, the original residual anomaly empirical isotropic covariance function was also used during the LSC prediction of the coefficients' corrections. To estimate such an isotropic covariance function empirically at a spherical distance ψ, the product sum average of pairs of anomaly values, relevant to pairs of points having spacing $\psi-\Delta\psi/2\leq\psi'\leq\psi+\Delta\psi/2$, was evaluated. Both $\Delta\psi$ and the ψ increment were chosen to be 2 minutes of arc and 100 covariance values (at 100 ψ values) were evaluated. Of course, such a function is dependent only on the spherical distances between pairs of stations, implying the invariance under a rotation of the data points group. An an-isotropic covariance function would be dependent on the positions of stations (Tscherning, 1999).

Figure (1) illustrates the input residual anomaly isotropic empirical covariance function and its associated fitted analytical function. The expression for the fitted covariance was as given by Eq.(6) with the following parameters

c = 5.961021,
R_b/R = 0.999916,
A = 145.518 $mgal^2$,
Δg variance at MSL = 760.410 $mgal^2$.

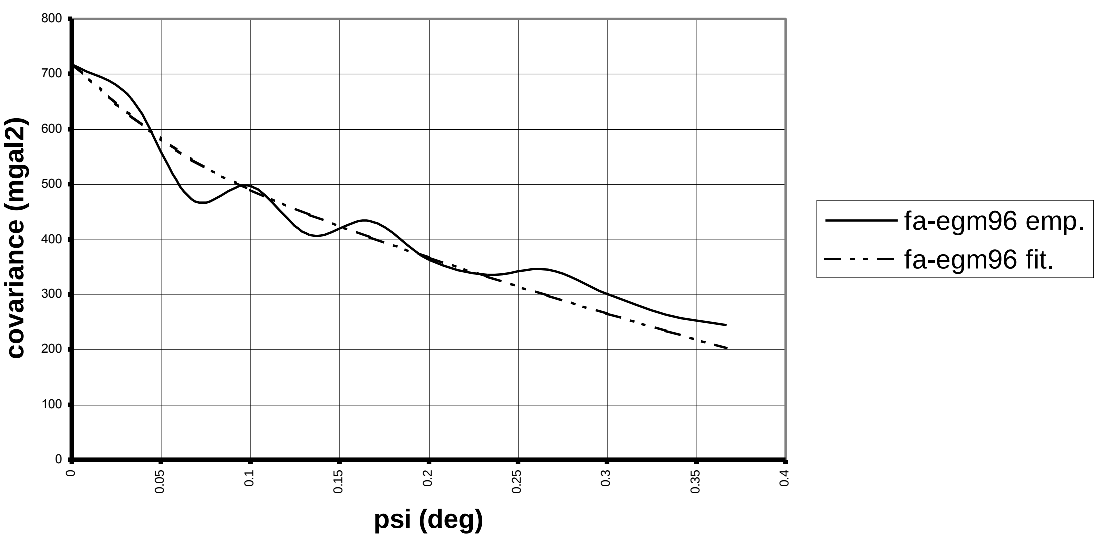

Figure (1): Free air residual anomaly empirical and fitted covariance function

The above covariance function parameters, the residual free air geoid data and the relevant error estimates were input in the LSC solution for the harmonic coefficients corrections along with their error estimates as follows

$$(GM/R).\Delta K_{ij} = C_{ij\,t}.(C_{tt} + E_{tt})^{-1} . l, \quad (7a)$$

$$E_{ij\,ij} = C_{ij\,ij} - C_{ij\,t}.(C_{tt} + E_{tt})^{-1} . C_{ij\,t}^{T}, \quad (7b)$$

with

$(GM/R).\Delta K_{ij}$	the estimated signal,
$C_{ij\,t}$	the cross-covariance vector between the signal and the (residual geoid) observations l,
C_{tt}	the covariance matrix of the (residual geoid) observations,
E_{tt}	the error variance-covariance matrix of the (residual geoid) observations,
l	the vector of (residual geoid) observations,
$E_{ij\,ij}$	the estimated error variance of the estimated signal,
$C_{ij\,ij}$	the signal variance as given by Eq.(3b).

8 Results

Recall that the solution for the corrections of the spherical harmonic coefficients (along with their error estimates) proceeded in a usual manner such as any remove-restore technique. The resulting corrections were in major cases significant. The solution, however, is very time consuming. The corrections were then added to the relevant original coefficients (Eq.(5c)), thus yielding the final locally refined coefficients, based on the Egyptian data. Figure (2) shows a graphical representation for the coefficients corrections. Figures (2a) and (2b) are simply gray scaled contour maps for the values of the corrections applied to the C and S-coefficients, respectively, from degree 2 and order zero to degree and order 360. This was done for the sake of an objective and detailed overview for the corrections received by the various spectral domains. The numerical values and the contour interval can be deduced from the associated gray scale, which implies that very significant values for the coefficients could be recovered. However, the corrections for the sectorial (m=n) and near sectorial ($m \approx n$) harmonic coefficients were negligible in most of the high degrees. These negligible corrections were more pronounced after degree and order 150. Thus, these terms could be very good represented by the EGM96 model or are not recoverable due to the local data geographical location. The first coefficients (from degree and order (2,0) to (10,10)) have received very small corrections, but within the number of significant figures of the original coefficients.

The EGM96EGR harmonic model was computationally removed from the start free air geoid grid, whose residual data were used for predicting the corrections. Table (2) shows a comparison between the statistics of the residual 0.5ºx0.5º geoid grid based on

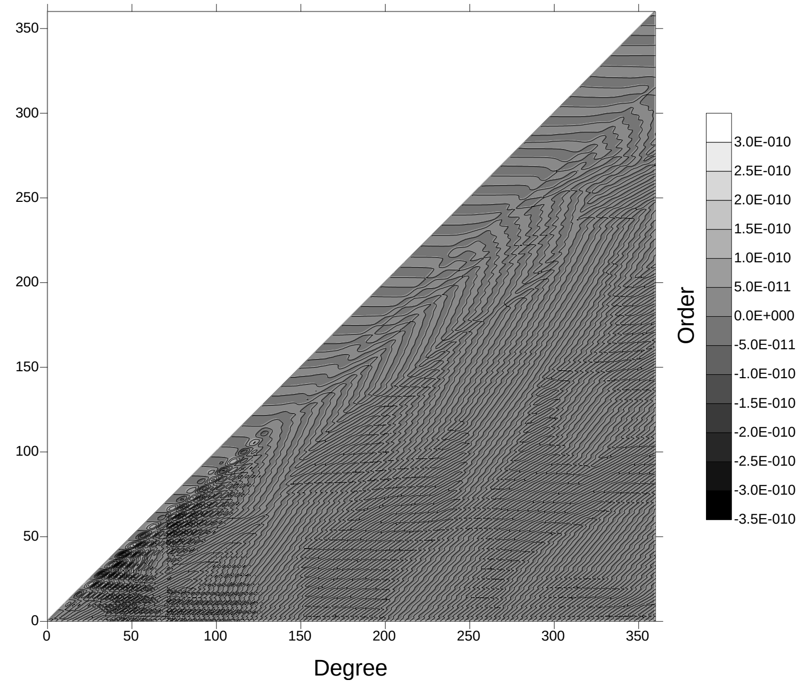

Figure (2a): Graphical representation of C_{ij} corrections (unitless)

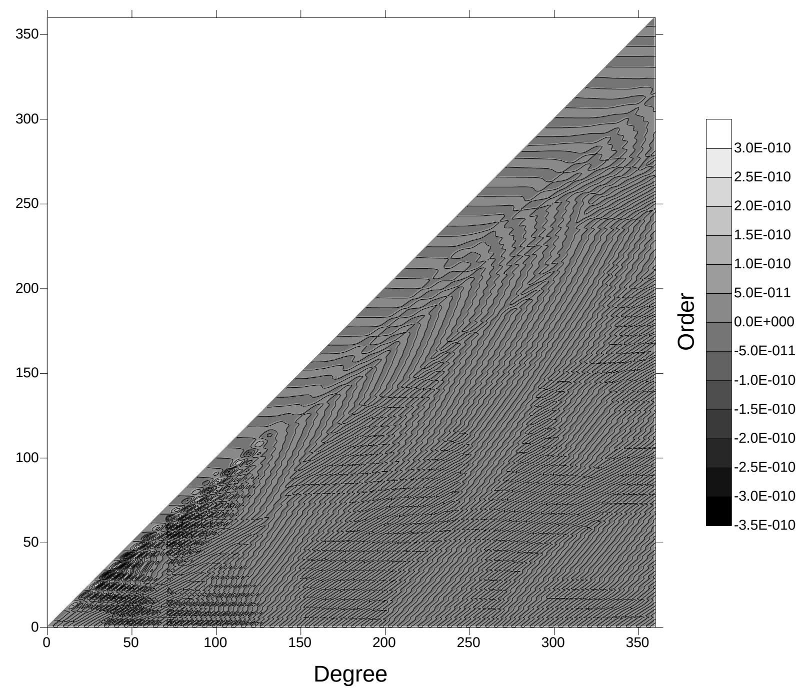

Figure (2b): Graphical representation of S_{ij} corrections (unitless)

Figure (2)

removing the EGM96EGR and those pertaining to the original harmonic model. From this table, one can notice the large amount of smoothness of the residual geoid data after suppressing the EGM96EGR. Besides having a smaller mean, a dramatic smoothing in terms of standard deviation manifests itself. The standard deviation and RMS of the residual geoid have decreased from 0.95 to 0.23 meter, thus having an improvement in geoid smoothing by about 76%. Also the minimum and maximum residuals were greatly reduced when using the refined model. The interpretation of the new residuals is twofold. On one hand, the EGM96EGR possesses a superior long to medium wavelength behavior over the original model. On the other hand, after suppressing the new refined model, there still exist short wavelength geoid residuals in the spectral domain higher than degree and order 360. This, in turn, ensures that the predicted corrections, and hence the refined harmonic coefficients, were not forced to absorb spectral information higher than the maximal degree of the EGM96 model.

The EGM96EGR model was also removed from the original scattered point gravity anomaly data. Table (3) shows the statistics of the original free air gravity anomaly data, the residual gravity anomaly data after removing the EGM96 model, and the residual data pertaining to the refined model. The gravity anomaly residual terrain effect was also taken into account, using a digital terrain model of Egypt computed by the authors (not published yet).

Table (2): Comparison between the statistics of the residual free air geoid grids (units: meters)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Geoid grid - EGM96	0.02	0.95	0.95	-2.66	5.26
Geoid grid – EGM96EGR	0.01	0.23	0.23	-1.11	1.10

Table (3): Comparison among the statistics of the raw and residual gravity anomalies (units: mgals)

Item	RTM	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity anomaly		-4.193	32.639	32.896	-144.270	227.247
Residual gravity anomaly (using EGM96)	No	-0.440	27.612	27.605	-155.505	206.327
	Yes	1.752	26.271	26.320	-160.802	200.094
Residual gravity anomaly (using EGM96EGR)	No	-0.087	16.642	16.636	-133.337	121.712
	Yes	2.105	15.923	16.056	-138.634	115.479

It is clear from Table (3) that the original model has smoothed the raw gravity anomaly by about 15%, in terms of standard deviation and RMS, whereas the refined model has about 49% smoothing effect. An improvement over the original model by about 40% has been achieved in the standard deviation and RMS of the residual free air gravity data, when using the EGM96EGR model. This relatively lower improvement, compared to 76% in geoid smoothing, is due to the fact that gravity anomaly still contains most of its power in the higher frequency spectral domain. The table shows also a greater decrease in the mean, minimum and maximum of the residual gravity data, due to the removal of the new refined model. On the other hand, due to the dominant moderate topographic variations in Egypt, the residual topographic effect is a minor factor in smoothing the gravity anomaly data, as can be noticed from the same table. Hence, concerning Egypt, the effect of incorporating the local data in a reference global harmonic model (about 40% RMS decrease), is much more pronounced than the effect of topographic effect (about 5% RMS decrease). Of course, the above remark concerning the guarantee that no high spectral information has been shifted to the lower degree terms in the refined model, is still valid for the case of gravity anomalies.

Figure (3) illustrates a plot of the corresponding residual gravity anomalies empirical covariance functions. This plot ascertains that the incorporation of the local data into the EGM96 model has a very great effect on the smoothness of the covariance function, compared to the minor role of the residual topographic effect. Inspecting the approximate first zeros of these functions, and using the $180^o/\psi^o$ theoretical rule of thumb, one could recognize that the covariance functions pertaining to the removal of the EGM96 has a first zero point tendency near $\psi \approx 2.14^o$, thus the original model has succeeded in suppressing only about 84 degrees from the raw data. This value nearly corresponds to the number of the satellite only degrees used in the combined global solutions for the original model. On the other hand, the refined model has elegantly removed about 360 degrees ($\psi \approx 0.5^o$) of the local data spectrum, which is a surprising result. Hence, the EGM96EGR could be considered to have achieved a toll improvement in the low frequency gravity field modeling in Egypt, according to the current available data.

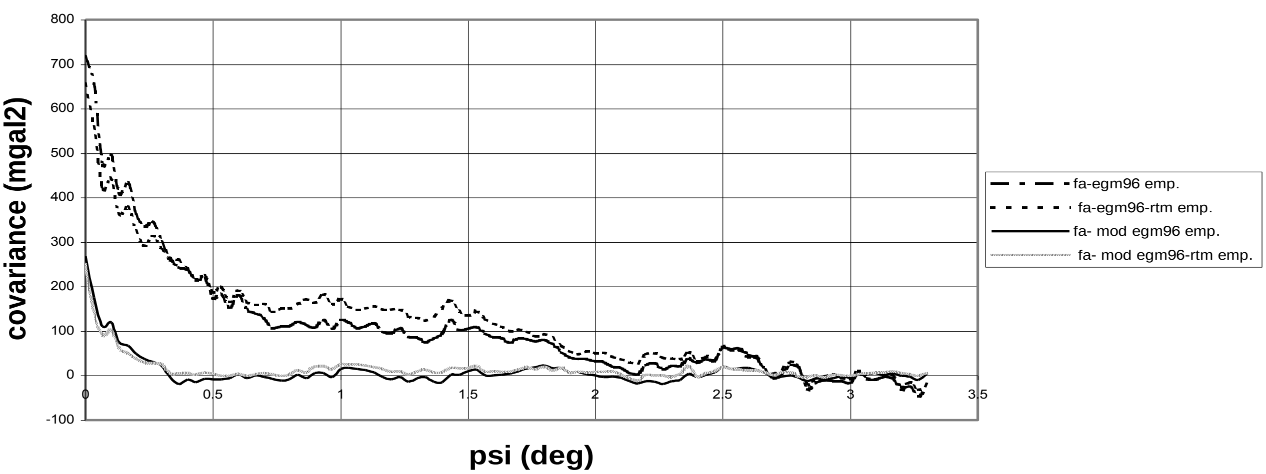

Figure (3): EGM96 and EGM96EGR related residual anomaly empirical covariance functions

9 Conclusions

From the current investigation, it is clear that the local corrections for the EGM96 global spherical harmonic model are numerically significant for most of the predicted terms. The EGM96EGR model is superior to the original one, regarding the smoothness of the respective residual data and its empirical covariance functions. The smoothness of the residual geoid height was improved by about 76% over the original model. The gravity anomaly smoothness was improved by about 40%. While the original model removal suppresses nearly as many degrees as the satellite only terms, the trend of the residual anomaly covariance function showed how elegantly the 360 lower degrees spectrum was approximately removed from the data after being referenced to the refined model. Thus, for the first time, regarding the Egyptian territory, we can lean on the resulting EGM96EGR, tailored to the Egyptian data, which is considered very capable of recovering the actual low-medium spectral information in Egypt.

It is recommended to use this EGM96EGR as a reference low degree field for future geoid solutions in Egypt. Moreover, this local corrections procedure could be used in the future to incorporate any updated gravity data base to the EGM96 model or into any new released version that may have no Egyptian data contribution.

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